EVALUATING DETERMINANTS

Definition: A determinant is a real number associated with a square array of real numbers and is indicated by enclosing the array between two vertical bars. For a matrix A, the corresponding determinant is designated as det(A) and is read "determinant of A."

$$\text{For } A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \text{ the } \det(A)=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

For 2 x 2 matrix, the determinant is calculated by multiplying the diagonal entries and then taking the product containing the element in the first row and first column and subtracting the other product from it.

$$\det(A)=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}=a_{11}a_{22}-a_{21}a_{12}$$

For a 3 x 3 matrix, a method called expanding by minors is used to find the determinant. A minor of a 3 x 3 determinant is found by selecting one row, then mentally crossing out the row and one column containing an element from that row.

For the determinant, $\det(A)=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$, the minor of a_{11} is $\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$.

Every minor has a sign associated with it, in a pattern which applies to any n x n determinant. The sign of a_{11} is always positive and alternates from there going both across and down, creating the pattern,

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}.$$

To calculate the determinant of the 3 x 3 matrix A,

$$\det(A)=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}=a_{11}(\text{minor of } a_{11})-a_{12}(\text{minor of } a_{12})+a_{13}(\text{minor of } a_{13})$$

$$=a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}-a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}+a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

The value of a determinant will be the same no matter which row or column is chosen to expand around.