FINDING THE COMMON DENOMINATOR

Definitions

- For a set of denominators, a Common Denominator is a number each denominator divides evenly into.
- The smallest common denominator is the Least Common Denominator (LCD).

Objective

- To Find a Common Denominator: Find a number whose prime factors include all the factors of the individual denominators. The number of possibilities is infinite.

> Example:
> Common denominators for the fractions $\frac{2}{5}$ and $\frac{1}{3}$ are $15, 30, 45, 60, 75, 90, \ldots$

- To Find the LCD: Look at the factorizations of the individual denominators. For each prime factor (i.e. 2, 3, 5, etc.), select the one with the largest exponent (i.e. from 2, 2^4, and 2^3, use only 2^4). Find the product of the prime numbers, each raised to the largest exponent. Using the LCD reduces the likelihood of errors because the numbers are as small as possible.

> Example:
> The denominators 8, 6 and 9 have the prime factorizations of $2^3, 2 \cdot 3$ and 3^2, respectively. Three is the largest exponent from 2^3 and 2. Two is the largest exponent from 3 and 3^2. The LCD will be $2^3 \cdot 3^2 = 8 \cdot 9 = 72$.

***Possible* Shortcuts to Finding the LCD**

- Select the largest denominator from the fractions in question. Test whether any of the other denominators will divide evenly into it. If they do, the largest denominator is the LCD.

> Example:
> The denominators $9, 36$ and 12 have an LCD of 36 because both 9 and 12 divide evenly into 36.

- If the denominators have no common factors, then multiply the denominators together.

> Example:
> The denominators $11, 9$ and 4 have the prime factorizations, respectively, of $11, 3^2$, and 2^2. With no common factors the LCD equals $11 \cdot 4 \cdot 9 = 396$.

A common denominator is only needed when adding or subtracting fractions.

See back for Methods of Finding Least Common Denominators.

METHODS FOR FINDING LEAST COMMON DENOMINATORS

Inspection Method

1) Find the prime factorizations of each of the denominators.
2) The LCD is the product of all the prime factors found in only one of denominators times each common prime number raised to the highest power it has in any of the denominators.

Example:

Find the LCD for $\frac{7}{15}, \frac{2}{9}$ and $\frac{16}{21}$.

$15 = 3 \cdot 5$
$9 = 3^2$
$21 = 3 \cdot 7$

The LCD=$5 \cdot 7 \cdot 3^2 = 315$ because the factors 5 and 7 each occur in one of the factorizations. The factor 3 occurs in all the denominators with its largest exponent being 2.

Chart Method

1) Find the prime factorizations of each of the denominators.
2) Enter the numbers and prime factorizations into a chart like the one shown below.
3) At the bottom of each prime number column, bring down the entry with the highest exponent.
4) The LCD is the product of all the entries in the bottom row.

Example:

Find the LCD for $\frac{11}{15}, \frac{9}{20}$ and $\frac{5}{24}$.

Factor denominators.
Put factors into the chart.
$15 = 3 \cdot 5$
$20 = 2^2 \cdot 5$
$24 = 2^3 \cdot 3$

Prime	2	3	5	
15=		$3 \cdot$	5	
20=	$2^2 \cdot$		5	
24=	$2^3 \cdot$	3		
LCD=	$2^3 \cdot$	$3 \cdot$	5	$= 120$

For factors of 2, 2^3 has the highest exponent. All the entries for 3 and for 5 are to the first power, so just 3 and 5 are brought down in their columns. Multiply the numbers in the last row.

Division Method

1) Divide the denominators by a prime number. (Starting with the smallest possible prime is good.) Instead of writing the answers above the numbers, write them below. If the number cannot be divided evenly, put a line through it and rewrite it in the line below. Repeat the process, always using a prime number as the divisor until the last line is a row of 1s.
2) The LCD is the product of all the divisors.

Example:

Find the LCD for $\frac{11}{15}, \frac{9}{20}$ and $\frac{5}{24}$.

$$\begin{array}{r|ccc}
2 & \cancel{15} & 20 & 24 \\
2 & \cancel{15} & 10 & 12 \\
2 & \cancel{15} & \cancel{5} & 6 \\
3 & 15 & \cancel{5} & 3 \\
5 & 5 & 5 & \cancel{1} \\
 & 1 & 1 & 1
\end{array}$$

$\text{LCD} = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$
$= 2^3 \cdot 3 \cdot 5 = 120$

See back for more information on Finding Least Common Denominators.